

DATE: - 23RD January, 2024
 CLASS: - SS 2^A
 TIME: - 8:00-9.20pm PERIOD: - 7th & 8th
 DURATION: - 80 Minutes
 SUBJECT: - Further Mathematics
 THEME: - Calculus
 UNIT TOPIC: - Differentiation
 LESSON TOPIC: - Differentiation of Transcendental functions
 SPECIFIC OBJECTIVES: - At the end of the lesson, the students should be able to;

- i. recall and manipulate trigonometrical ratios
- ii. prove the differentiation of given trigonometrical ratios
- iii. discuss the standard table of the differentiation of given trigonometric functions
- iv. apply the formulas from the standard table on the differentiation of circular function
- v. respond to questions on the differentiation of transcendental functions

INSTRUCTIONAL RESOURCES: - Illustrative chart.

PRESENTATION

STEP 1: Identification of prior ideas

MODE: - Whole

Teachers Activities: The teacher drills the student on SOH/CAH/TOA and its manipulation.

Students Activities: - The students respond and listen to the contribution of the teacher.

$$\sin = \frac{\text{opp.}}{\text{hyp.}}, \cos = \frac{\text{adj}}{\text{hyp.}}, \tan = \frac{\text{opp.}}{\text{adj.}}, \text{Cosec} = \frac{1}{\sin}, \text{Sec} = \frac{1}{\cos}, \text{Cot} = \frac{1}{\tan}$$

The derivatives of other circular functions can be obtained from the derivatives of the functions of sin x, cos x and tan x. It is important to note that it is assumed that x is measured in radian (180° being equal to 2π radian). It is also important to observe that when θ is small,

$$\text{So that } \theta < \frac{\pi}{18}, \text{ then } \sin \theta = \theta, \cos \theta = 1, \tan \theta = \theta$$

$$\text{Thus, as } \delta x \rightarrow 0, \sin \delta x \rightarrow \delta x, \cos \delta x \rightarrow 1 \text{ and } \tan \delta x \rightarrow \delta x.$$

STEP 2: - Exploration

MODE: - Whole

Teachers Activities: - The teacher illustrates differentiation of trigonometric functions to the class using a chart.

Students Activities: - The class listens, ask relevant questions and copied their notes.

1. If $y=f(x) = \sin x$

$$\begin{aligned} \frac{\delta y}{\delta x} &= \frac{f(x+\delta x)-f(x)}{\delta x} \\ &= \frac{\sin(x+\delta x)-\sin x}{\delta x} \\ &= \frac{\sin x \cos \delta x + \cos x \sin \delta x - \sin x}{\delta x} \end{aligned}$$

(Note: $\sin (A+B) = \sin A \cos B + \cos A \sin B$)

$$\begin{aligned} \text{As } \delta x &\rightarrow 0 \\ \frac{\delta y}{\delta x} &\rightarrow \frac{\sin x + \cos x \cdot \delta x - \sin x}{\delta x} = \frac{\delta x \cos x}{\delta x} \\ &= \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \cos x \end{aligned}$$

Therefore we say that the derivative of $\sin x$ is obtained from the first principle.

STEP 3: - Discussion

MODE: - Whole

Teachers Activities: - The teacher completes the standard table of the differentiation of given trigonometric functions

Students Activities: - The students identify and discuss the differentiation of the given trigonometric functions from the standard table.

The teacher presents a chart on the standard table for the differentiation of circular functions.

y	$\frac{\delta y}{\delta x}$
K (constant)	0
Sin x	Cos x
Cos x	-Sin x
tan x	sec ² x
Sec x	Sec x tan x
Cosec x	Cosec x cot x
Cot x	-cosec ² x

STEP 4: - Application

MODE: - Whole

Teachers Activities: - The teacher solves examples using the standard formula table.

Students Activities: - The students listen as the teacher applies the differentiation of circular function

The teacher solves examples using the formula.

1. If $y = \sin 3x$

$$Y = \sin u, \quad \frac{\delta y}{\delta u} = \cos u$$

$$U = 3x, \quad \frac{\delta u}{\delta x} = 3$$

$$\frac{\delta y}{\delta x} = \frac{\delta y}{\delta u} \times \frac{\delta u}{\delta x}$$

$$\frac{\delta y}{\delta x} = \cos u \cdot 3 = 3\cos 3x \quad \text{e.t.c.}$$

$$2\sin x \cos x$$

2. Differentiate the following with respect to x .

- a. $3x\cos x$ b. $x^2/\sin x$ c. $\sec x \tan x$

Solution:

$$\text{a. } \frac{d}{dx}(3x\cos x) = 3x \frac{d}{dx} \cos x + \cos x \frac{d}{dx} 3x = -3x\sin x + 3\cos x = 3\cos x - 3x\sin x = 3(\cos x - x\sin x).$$

$$\text{b. } x^2/\sin x = \frac{d}{dx}(x^2/\sin x) = [\sin x \frac{d}{dx}(x^2) - x^2 \frac{d}{dx} \sin x] \div \sin^2 x.$$

$$\text{c. } \sec x \tan x \Rightarrow \sec x \frac{d}{dx} \tan x + \tan x \frac{d}{dx} \sec x = \sec x(\sec^2 x + \tan x(\sec x \tan x))$$

$$\therefore \frac{dx}{dy} = \sec x(\sec^2 x + \tan^2 x)$$

STEP 5: - Evaluation

MODE: - Whole

Teachers Activities: - The teacher drill the students on questions related to the lesson.

1. Briefly discuss the basics of circular function?
2. Differentiate $\cos x$ and $\tan x$.
3. Draw the standard table for the differentiation of circular functions.
4. Solve for $y=x^2 + 3\sin x$

Students Activities: - The entire class responds to the class exercise.

CONCLUSION: - The teacher marks the class exercise and writes correction on the chalkboard.

ASSIGNMENT: - Exercise 4b,nos 4&5,Further Mathematics for senior secondary schools.

.REFEREENCE BOOK: - 1. Engineering Mathematics by K Strod.
2. Pure Mathematics by Backhouse .